

Tilburg University

An upper and a lower bound for the distance of a manifold to a nearby point

Paardekooper, M.H.C.

Publication date:
1988

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

Paardekooper, M. H. C. (1988). *An upper and a lower bound for the distance of a manifold to a nearby point*. (Research memorandum / Tilburg University, Department of Economics; Vol. FEW 321). Unknown Publisher.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

CBM
R

7626
1988
321

UNIVERSITY
HOLLAND
UNIVERSITEIT
BRABANT

POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS



DEPARTMENT OF ECONOMICS
RESEARCH MEMORANDUM

R4626

no.

321

(1988?)



AN UPPER AND A LOWER BOUND FOR THE
DISTANCE OF A MANIFOLD TO A NEARBY
POINT

M.H.C. Paardekooper

FEW 321

F 83
518 316

AN UPPER AND A LOWER BOUND FOR THE DISTANCE OF A MANIFOLD

TO A NEARBY POINT

M.H.C. Paardekooper

Department of Econometrics
Tilburg University
Tilburg, The Netherlands

This research is part of the VF-program "Econometrie van Micro-economische gedragsmodellen", which has been approved by the Netherlands Ministry of Education and Sciences.

ABSTRACT

A generalization for underdetermined systems of the wellknown Newton-Kantorovich theorem gives bounds for the distance of a point, say O , in Hilbert space X to a nearby manifold $S = \{x \in X \mid f(x) = 0\}$. Here $f: X \rightarrow Y$ is a differentiable mapping such that $Df(O)$ is surjective; $f(O)$, Df together with right inverse $Df(O)^+$ satisfies typical Kantorovich-like conditions. Analysis in the normal space at O of $\tilde{S} = \{x \in X \mid f(x) = f(O)\}$ gives an upperbound of $d(O, S)$. Furthermore the Kantorovich conditions effect S to be locally in a convex cone. The distance of O to that cone gives a lowerbound of $d(O, S)$.

1. INTRODUCTION

The purpose of this paper is to derive bounds, both upper bounds as lower bounds, for the distance of a manifold to a nearby point.

The starting point and main tool in the construction of the bounds is the classical Newton-Kantorovich theorem.

THEOREM 0.[2]. Let Y, Z be Banach spaces. Let $B(0, r)$ be an open ball in Banach space Z and let be $\varphi: B(0, r) \subset Z \rightarrow Y$ Frechet differentiable on $B(0, r)$ with

$$\|D\varphi(x) - D\varphi(y)\| \leq L \|x - y\|, \quad x, y \in B(0, r). \quad (1.1)$$

Assume that $D\varphi(0)^{-1} \in \mathcal{L}(Z, Y)$ exists,

$$\|D\varphi(0)^{-1}\| \leq \lambda^{-1}, \quad \|D\varphi(0)^{-1} \varphi(0)\| = \tilde{\gamma} \leq \gamma, \quad \kappa := L \gamma \lambda^{-1} < \frac{1}{2}$$

and

$$M := \lambda(1 - \sqrt{1 - 2\kappa})/L < r. \quad (1.2)$$

Then the equation $\varphi(x) = 0$ has a solution $z \in B(0, M) \subset Z$ and z is a unique zero of φ in $B(0, \rho_1) \subset Z$, where

$$\rho_1 = \lambda(1 + \sqrt{1 - 2\kappa})/L \quad \square$$

For the formulation of the distance problem in section two we consider the Hilbert spaces X and Y and we investigate $f: B(0, r) \subset X \rightarrow Y$, a Frechet differential mapping. We assume that

(i) $A := Df(0) \in \mathcal{L}(X, Y)$ is surjective and $\|A^+\| \leq \lambda^{-1}$, where $A^+ \in \mathcal{L}(X, Y)$ is the right inverse of A ,

$$(ii) \quad \|Df(x) - Df(y)\| \leq L \|x - y\|, \quad x, y \in B(0, r), \quad (1.4)$$

$$(iii) \quad \|A^+ f(0)\| = \tilde{\gamma} \leq \gamma, \quad (1.5)$$

$$(iv) \quad \kappa = L \gamma \lambda^{-1} < \frac{1}{2}, \quad (1.6)$$

and

$$(v) \quad M := \lambda(1 - \sqrt{1 - 2\kappa})/L < r. \quad (1.7)$$

We use the following notation:

$$S = \{x \in B(0, r) \mid f(x) = 0\} \quad (1.8)$$

and

$$\tilde{S} = \{x \in B(0, r) \mid f(x) = f(0)\}. \quad (1.9)$$

N_1 denotes $\text{Ker}(A)$ being the tangent space of \tilde{S} in 0. Let be N_2 the orthogonal complement of N_1 .

Analysis in the normal space N_2 at 0 of \tilde{S} leads to an upperbound of $d(0, S)$, theorem 1. The Kantorovich condition (1.6) effects S to be locally in a convex cone. Theorem 2 gives the distance of 0 to that cone as a lower bound of $d(0, S)$.

This general approach leads to a manageable method to determine sharp error bounds for an approximate solution of an undetermined system.

2. BOUNDS FOR $d(0, S)$

THEOREM 1. The mapping $f: B(0, r) \subset X \rightarrow Y$ described in the introduction has a zero z in $B(0, M) \cap N_2$; z is the unique zero of $\varphi = f|_{N_2}$ in $B(0, \rho_1) \cap N_2$ where

$$\rho_1 = \lambda(1 + \sqrt{1-2\kappa})/L. \quad (2.1)$$

PROOF. The surjectivity of A implies that the restriction $A|_{N_2}$ is bijective. Its inverse is also continuous and equals the right inverse $A^+ = A^* (AA^*)^{-1}$ of A [1].

Let be $\varphi = f|_{N_2}$. This mapping $\varphi: N_2 \cap B(0, r) \rightarrow Y$ satisfies the conditions of the Newton-Kantorovich theorem 0 formulated above. Hence $\varphi(x) = 0$ has a solution z in $B(0, M) \cap N_2$ and z is the unique zero of φ in $B(0, \rho_1) \cap N_2$. \square

LEMMA 1. For the zero z of $\varphi = f|_{N_2}$ in $B(0, M) \cap N_2$ holds

$$\beta := \|z\| \geq \rho_2, \quad (2.2)$$

where

$$\rho_2 = \lambda(-1 + \sqrt{1+2\tilde{\kappa}})/L, \quad (2.3)$$

with $\tilde{\kappa} = \tilde{\gamma}\lambda^{-1}$.

PROOF. Since Df is Lipschitz continuous on $B(0, r)$ we have $\|f(z) - f(0) - Az\| \leq \frac{1}{2} L \beta^2$ and consequently [3]

$$\tilde{\gamma} = \|A^+ f(0)\| = \|z + A^+(f(z) - f(0) - Az)\| \leq \beta + \frac{1}{2} L \beta^2 \lambda^{-1},$$

for $z = A^+ Az$ as follows from $z \in R(A^*)$.

Hence the positive zero ρ_2 of the quadratic function $t \rightarrow \frac{1}{2} L \lambda^{-1} t^2 + t - \tilde{\gamma}$ is majorized by β . That proves (2.3). \square

REMARK. As a consequence of (1.7) we get $\beta < M < 2\gamma$. Hence

$$L\beta < 2L\gamma < \lambda. \quad \square \quad (2.4)$$

In the sequel we use the following notation

$$V = \{x \in X \mid \|x\| < \beta\}, P(x) := Df(x)|_{N_1}, Q(x) := Df(x)|_{N_2}, x \in V, \quad (2.5)$$

$$\alpha = \frac{L\beta}{\lambda - L\beta}, \quad (2.6)$$

where, as above $\beta = \|z\|$.

LEMMA 2. $Q(x)$ is regular for $x \in V$ and

$$\|Q(x)^{-1} P(x)\| \leq \alpha, \quad x \in V. \quad (2.7)$$

PROOF. Let be $x \in V$ and $y = y_1 + y_2$, $y_i \in N_i$, $i = 1, 2$. Then $Df(x)y = P(x)y_1 + Q(x)y_2$. Since $P(0) = 0$, $\|Px\| \leq \|Df(x) - Df(0)\| \leq L\beta$, $x \in V$. Similarly $\|Q(x) - Q(0)\| \leq \|Df(x) - Df(0)\| \leq L\beta$, $x \in V$. Since $Q(0) = D\varphi(0)$,

$$\|(Q(x) - Q(0)) Q(0)^{-1}\| \leq L\beta\lambda^{-1} < 1$$

as follows from $\beta < M$ and (2.4). This implies that

$$Q(x) = (I + (Q(x) - Q(0)) Q(0)^{-1}) Q(0)$$

is invertible for each $x \in V$ and

$$\|Q(x)^{-1}\| \leq \|Q(0)^{-1}\| (1 - L\beta\lambda^{-1})^{-1} \leq (\lambda - L\beta)^{-1}.$$

Hence $\|Q(x)^{-1} P(x)\| \leq L\beta(\lambda - L\beta)^{-1} = \alpha$. □

For reasons of shortness we define

$$W = \{x = x_1 + x_2 \in X \mid \alpha \|x_1\| + \|x_2\| < \beta, x_i \in N_i, i = 1, 2\} \quad (2.8)$$

and with $w = w_1 + w_2 \in X$, $w_i \in N_i$, $i = 1, 2$ (2.9)

$$K(w) = \{(1-\tau)w_1 + x_2 \mid \tau \in [0, 1], x_2 \in N_2, \|x_2 - w_2\| \leq \alpha \|w_1\| \tau\}.$$

The lines along which the proof of theorem 2 will be given can be explained with a figure.

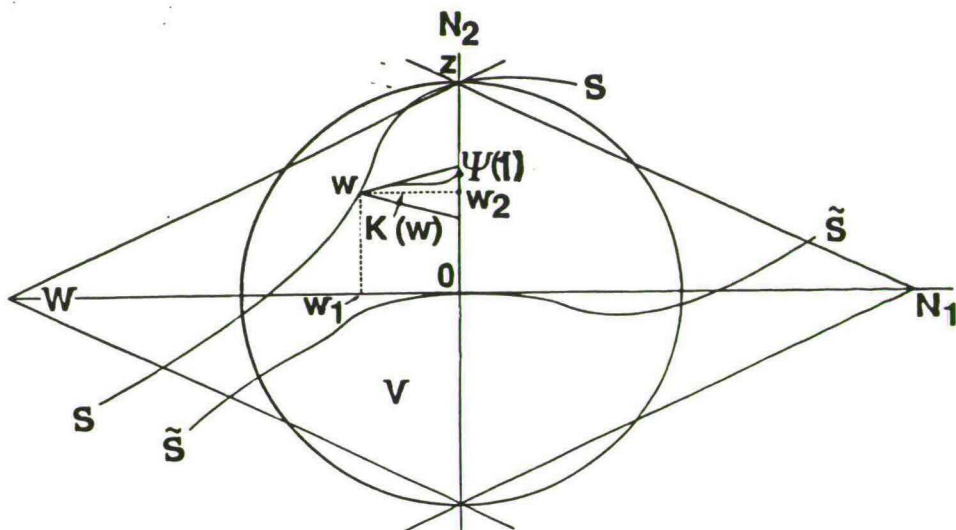


fig. 1. $w \in S \cap W \cap V$ contradicts $S \cap V \cap N_2 = \emptyset$.

In lemma 3 we prove that $w \in W \cap V$ implies $K(w) \subset W \cap V$. In lemma 4 indirectly we prove that $w \in W \cap V$ implies $f(w) \neq 0$. So $S \subset V^c \cup W^c$ and $d(0, S) \geq d(0, W^c)$. In this manner we get $m := d(0, W^c)$ as an upper bound for the distance of 0 to the manifold S .

LEMMA 3. If $w = w_1 + w_2 \in W \cap V$ then $K(w) \subset V \cap W$.

PROOF. Let be $x = x_1 + x_2 \in K(w)$. Then there exist $\epsilon, \tau \in [0, 1]$ and a unit vector $v \in N_2$ such that $x_1 = (1-\tau)w_1$ and $x_2 = w_2 + \epsilon\alpha|w_1|\tau v$. Thus

$$|x|^2 = (1-\tau)^2|w_1|^2 + |w_2 + \epsilon\alpha|w_1|\tau v|^2 \leq (1-\tau)^2|w_1|^2 + (|w_2| + \alpha\tau|w_1|)^2 = g(\tau).$$

Now $g(0) = |w|^2 < \beta^2$ and $g(1) = (|w_2| + \alpha|w_1|)^2 < \beta^2$ for $w \in V$ and $w \in W$ respectively. So $x \in V$.

Similarly we have $\alpha|x_1| + |x_2| \leq \alpha|w_1|(1-(1-\epsilon)\tau) + |w_2| < \beta$. Thus also $x \in W$. □

LEMMA 4. The function f has no zero in $W \cap V$.

PROOF. Assume $w = w_1 + w_2 \in W \cap V$, $w_1 \in N_1$, $i = 1, 2$ and $f(w) = 0$. Define a function G as follows

$$(\tau, x_2) \rightarrow G(\tau, x_2) = f((1-\tau)w_1 + x_2), \quad x_2 \in N_2, \quad (1-\tau)^2|w_1|^2 + |x_2|^2 < r^2.$$

Then $G(0, w_2) = 0$ and the derivative $D_2 G(0, w_2)$ of G in $(0, w_2)$ with respect to x_2 equals $Q(w)$. By lemma two $Q(w)$ is regular. According to the implicit function theorem there exists a $\delta > 0$ and a differentiable function $\psi: (-\delta, \delta) \rightarrow N_2 \cap V$ such that $\psi(0) = w_2$ and for $\tau \in (-\delta, \delta)$ holds

$$G(\tau, \psi(\tau)) = 0, \quad D\psi(\tau) = -D_2 G(\tau, \psi(\tau))^{-1} D_1 G(\tau, \psi(\tau))$$

where D_1 and D_2 denote differentiation with respect to τ and x_2 respectively. Since

$$D_1 G(\tau, x_2) = -P((1-\tau)w_1 + x_2)w_1, \quad D_2 G(\tau, x_2) = Q((1-\tau)w_1 + x_2)$$

we have

$$|D\psi(\tau)| \leq \|Q((1-\tau)w_1 + \psi(\tau))^{-1} P((1-\tau)w_1 + \psi(\tau))\| \|w_1\| \leq \alpha \|w_1\|, \quad |\tau| < \delta,$$

as follows from lemma 2. Consequently

$$\|\psi(\tau) - w_2\| = \left\| \int_0^\tau D\psi(\sigma) d\sigma \right\| \leq \alpha \|w_1\| \tau, \quad 0 < \tau < \delta.$$

Hence $(1-\tau)w_1 + \psi(\tau) \in K(w)$ if $\tau \in [0, \delta) \subset [0, 1]$. If $\tau_1, \tau_2 \in [0, \delta)$ then $\|\psi(\tau_1) - \psi(\tau_2)\| \leq \alpha \|w_1\| |\tau_1 - \tau_2|$ which implies, by the Cauchy criterion, that $\tilde{w} = \lim_{\tau \uparrow \delta} ((1-\tau)w_1 + \psi(\tau))$ exists in the closed $K(w)$ and thus $\tilde{w} \in V \cap W$ as follows from lemma 3. So the function ψ can be prolonged and extended until τ equals 1, i.e. $G(1, \psi(1)) = 0$. Thus $f(\psi(1)) = 0$. That means $\varphi(\psi(1)) = 0$, with $\psi(1) \in V \cap N_2$ and $\varphi = f|_{N_2}$. This contradicts theorem 1. Hence $w \in W \cap V$ implies $f(w) \neq 0$. \square

THEOREM 2. Let $f: B(0, r) \subset X \rightarrow Y$ satisfy the conditions given in the introduction and let be

$$g(\tau) := \tau(\lambda/L - \tau)(\tau^2 + (\lambda/L - \tau)^2)^{-\frac{1}{2}}, \quad 0 < \tau < \frac{\lambda}{L}. \quad (2.10)$$

Then

$$d(0, S) \geq m := \begin{cases} g(M) & , \frac{1}{4} \sqrt{3} \leq \kappa < \frac{1}{2} \text{ and } \sqrt{1-2} + \frac{1}{2}(1-2\kappa) \leq \tilde{\kappa} \leq \kappa \\ g(\rho_2) & , 0 < \kappa < \frac{1}{2} \text{ and } \tilde{\kappa} \leq \min\{\kappa, \sqrt{1-2\kappa} + \frac{1}{2}(1-2\kappa)\} \end{cases} \quad (2.11)$$

where M and ρ_2 as given in (1.7) and (2.3) respectively.

PROOF. With simple computations we find

$$d(0, W^C) = \beta(1+\alpha^2)^{-\frac{1}{2}} = g(\beta) < \beta,$$

where α and β are given in (2.6) and (2.2) respectively. So

$$d(0, S) \geq d(0, W^C \cup V^C) = g(\beta).$$

It is easy to see that $\tau = \frac{1}{2} \lambda/L$ is the axis of symmetry of the graph of g . The function g increases on $(0, \frac{1}{2} \lambda/L]$ from 0 until its maximum $\frac{1}{4} \lambda \sqrt{2}/L$ and decreases on $[\frac{1}{2} \lambda/L, \lambda/L)$ to zero. Since $\rho_2 \leq \beta \leq M$ as we know from theorem 1 and lemma 1,

$$d(0, S) \geq m := \min\{g(\rho_2), g(M)\}.$$

The symmetry of g implies that

$$m = g(M) \Leftrightarrow M \geq \frac{1}{2} \lambda/L \text{ and } \rho_2 \geq \lambda/L - M \quad (2.12)$$

and

$$m = g(\rho_2) \Leftrightarrow M < \frac{1}{2} \lambda/L \text{ or } (M \geq \frac{1}{2} \lambda/L \text{ and } \rho_2 < \lambda/L - M). \quad (2.13)$$

With (1.7) we find $M \geq \frac{1}{2} \lambda/L$ iff $\kappa \geq \frac{3}{8}$ and with (2.3) we get that $\rho_2 \geq \lambda/L - M$ iff $\sqrt{1-2\kappa} + \frac{1}{2}(1-2\kappa) \leq \tilde{\kappa} \leq k$. Since $\kappa \geq \sqrt{1-2\kappa} + \frac{1}{2}(1-\kappa)$ for $\kappa \geq \frac{1}{4} \sqrt{3}$, (2.11) can be concluded. \square

COROLLARY. If $\gamma = \tilde{\gamma}$, i.e. $\kappa = \tilde{\kappa}$, then

$$m = d(0, S) \geq \begin{cases} g(M) & , \frac{1}{4} \sqrt{3} \leq \kappa < \frac{1}{2} \\ g(\rho_2) & , \kappa < \frac{1}{4} \sqrt{3} \end{cases} \quad (2.14)$$

PROOF. The two conditions (2.12) and (2.14) lead to the two cases of (2.14) with the same means as in the theorem. \square

3. REFERENCES

1. Aubin, J.B.; Applied Functional Analysis, Wiley, New York, xv + 423 pp., 1979.
2. Gragg, W.B. and R.A. Tapia; Optimal Error Bounds for the Newton-Kantorovich Theorem, SIAM J. Numer. Anal., 17, 1980, pp. 883-893.
3. Tapia, R.A.; Differentiation and Integration of Nonlinear Operators, in Nonlinear Functional Analysis and Applications, L.B. Rall, ed., Academic Press, New York, 1971, pp. 45-103.

IN 1987 REEDS VERSCHENEN

- 242 Gerard van den Berg
Nonstationarity in job search theory
- 243 Annie Cuyt, Brigitte Verdonk
Block-tridiagonal linear systems and branched continued fractions
- 244 J.C. de Vos, W. Vervaat
Local Times of Bernoulli Walk
- 245 Arie Kapteyn, Peter Kooreman, Rob Willemse
Some methodological issues in the implementation
of subjective poverty definitions
- 246 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel
Sampling for Quality Inspection and Correction: AOQL Performance
Criteria
- 247 D.B.J. Schouten
Algemene theorie van de internationale conjuncturele en structurele
afhankelijkheden
- 248 F.C. Bussemaker, W.H. Haemers, J.J. Seidel, E. Spence
On (v,k,λ) graphs and designs with trivial automorphism group
- 249 Peter M. Kort
The Influence of a Stochastic Environment on the Firm's Optimal Dyna-
mic Investment Policy
- 250 R.H.J.M. Gradus
Preliminary version
The reaction of the firm on governmental policy: a game-theoretical
approach
- 251 J.G. de Gooijer, R.M.J. Heuts
Higher order moments of bilinear time series processes with symmetri-
cally distributed errors
- 252 P.H. Stevers, P.A.M. Versteijne
Evaluatie van marketing-activiteiten
- 253 H.P.A. Mulders, A.J. van Reeken
DATAAL - een hulpmiddel voor onderhoud van gegevensverzamelingen
- 254 P. Kooreman, A. Kapteyn
On the identifiability of household production functions with joint
products: A comment
- 255 B. van Riel
Was er een profit-squeeze in de Nederlandse industrie?
- 256 R.P. Gilles
Economies with coalitional structures and core-like equilibrium con-
cepts

- 257 P.H.M. Ruys, G. van der Laan
Computation of an industrial equilibrium
- 258 W.H. Haemers, A.E. Brouwer
Association schemes
- 259 G.J.M. van den Boom
Some modifications and applications of Rubinstein's perfect equilibrium model of bargaining
- 260 A.W.A. Boot, A.V. Thakor, G.F. Udell
Competition, Risk Neutrality and Loan Commitments
- 261 A.W.A. Boot, A.V. Thakor, G.F. Udell
Collateral and Borrower Risk
- 262 A. Kapteyn, I. Woittiez
Preference Interdependence and Habit Formation in Family Labor Supply
- 263 B. Bettonvil
A formal description of discrete event dynamic systems including perturbation analysis
- 264 Sylvester C.W. Eijffinger
A monthly model for the monetary policy in the Netherlands
- 265 F. van der Ploeg, A.J. de Zeeuw
Conflict over arms accumulation in market and command economies
- 266 F. van der Ploeg, A.J. de Zeeuw
Perfect equilibrium in a model of competitive arms accumulation
- 267 Aart de Zeeuw
Inflation and reputation: comment
- 268 A.J. de Zeeuw, F. van der Ploeg
Difference games and policy evaluation: a conceptual framework
- 269 Frederick van der Ploeg
Rationing in open economy and dynamic macroeconomics: a survey
- 270 G. van der Laan and A.J.J. Talman
Computing economic equilibria by variable dimension algorithms: state of the art
- 271 C.A.J.M. Dirven and A.J.J. Talman
A simplicial algorithm for finding equilibria in economies with linear production technologies
- 272 Th.E. Nijman and F.C. Palm
Consistent estimation of regression models with incompletely observed exogenous variables
- 273 Th.E. Nijman and F.C. Palm
Predictive accuracy gain from disaggregate sampling in arima - models

- 274 Raymond H.J.M. Gradus
The net present value of governmental policy: a possible way to find the Stackelberg solutions
- 275 Jack P.C. Kleijnen
A DSS for production planning: a case study including simulation and optimization
- 276 A.M.H. Gerards
A short proof of Tutte's characterization of totally unimodular matrices
- 277 Th. van de Klundert and F. van der Ploeg
Wage rigidity and capital mobility in an optimizing model of a small open economy
- 278 Peter M. Kort
The net present value in dynamic models of the firm
- 279 Th. van de Klundert
A Macroeconomic Two-Country Model with Price-Discriminating Monopolists
- 280 Arnoud Boot and Anjan V. Thakor
Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing
- 281 Arnoud Boot and Anjan V. Thakor
Appendix: "Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing"
- 282 Arnoud Boot, Anjan V. Thakor and Gregory F. Udell
Credible commitments, contract enforcement problems and banks: intermediation as credibility assurance
- 283 Eduard Ponds
Wage bargaining and business cycles a Goodwin-Nash model
- 284 Prof.Dr. hab. Stefan Mynarski
The mechanism of restoring equilibrium and stability in polish market
- 285 P. Meulendijks
An exercise in welfare economics (II)
- 286 S. Jørgensen, P.M. Kort, G.J.C.Th. van Schijndel
Optimal investment, financing and dividends: a Stackelberg differential game
- 287 E. Nijssen, W. Reijnders
Privatisering en commercialisering; een oriëntatie ten aanzien van verzelfstandiging
- 288 C.B. Mulder
Inefficiency of automatically linking unemployment benefits to private sector wage rates

- 289 M.H.C. Paardekooper
A Quadratically convergent parallel Jacobi process for almost diagonal matrices with distinct eigenvalues
- 290 Pieter H.M. Ruys
Industries with private and public enterprises
- 291 J.J.A. Moors & J.C. van Houwelingen
Estimation of linear models with inequality restrictions
- 292 Arthur van Soest, Peter Kooreman
Vakantiebestemming en -bestedingen
- 293 Rob Alessie, Raymond Gradus, Bertrand Melenberg
The problem of not observing small expenditures in a consumer expenditure survey
- 294 F. Boekema, L. Oerlemans, A.J. Hendriks
Kansrijkheid en economische potentie: Top-down en bottom-up analyses
- 295 Rob Alessie, Bertrand Melenberg, Guglielmo Weber
Consumption, Leisure and Earnings-Related Liquidity Constraints: A Note
- 296 Arthur van Soest, Peter Kooreman
Estimation of the indirect translog demand system with binding non-negativity constraints

IN 1988 REEDS VERSCHENEN

- 297 Bert Bettonvil
Factor screening by sequential bifurcation
- 298 Robert P. Gilles
On perfect competition in an economy with a coalitional structure
- 299 Willem Selen, Ruud M. Heuts
Capacitated Lot-Size Production Planning in Process Industry
- 300 J. Kriens, J.Th. van Lieshout
Notes on the Markowitz portfolio selection method
- 301 Bert Bettonvil, Jack P.C. Kleijnen
Measurement scales and resolution IV designs: a note
- 302 Theo Nijman, Marno Verbeek
Estimation of time dependent parameters in linear models
using cross sections, panels or both
- 303 Raymond H.J.M. Gradus
A differential game between government and firms: a non-cooperative
approach
- 304 Leo W.G. Strijbosch, Ronald J.M.M. Does
Comparison of bias-reducing methods for estimating the parameter in
dilution series
- 305 Drs. W.J. Reijnders, Drs. W.F. Verstappen
Strategische bespiegelingen betreffende het Nederlandse kwaliteits-
concept
- 306 J.P.C. Kleijnen, J. Kriens, H. Timmermans and H. Van den Wildenberg
Regression sampling in statistical auditing
- 307 Isolde Woittiez, Arie Kapteyn
A Model of Job Choice, Labour Supply and Wages
- 308 Jack P.C. Kleijnen
Simulation and optimization in production planning: A case study
- 309 Robert P. Gilles and Pieter H.M. Ruys
Relational constraints in coalition formation
- 310 Drs. H. Leo Theuns
Determinanten van de vraag naar vakantiereizen: een verkenning van
materiële en immateriële factoren
- 311 Peter M. Kort
Dynamic Firm Behaviour within an Uncertain Environment
- 312 J.P.C. Blanc
A numerical approach to cyclic-service queueing models

- 313 Drs. N.J. de Beer, Drs. A.M. van Nunen, Drs. M.O. Nijkamp
Does Morkmon Matter?
- 314 Th. van de Klundert
Wage differentials and employment in a two-sector model with a dual
labour market
- 315 Aart de Zeeuw, Fons Groot, Cees Withagen
On Credible Optimal Tax Rate Policies
- 316 Christian B. Mulder
Wage moderating effects of corporatism
Decentralized versus centralized wage setting in a union, firm,
government context
- 317 Jörg Glombowski, Michael Krüger
A short-period Goodwin growth cycle
- 318 Theo Nijman, Marno Verbeek, Arthur van Soest
The optimal design of rotating panels in a simple analysis of
variance model
- 319 Drs. S.V. Hannema, Drs. P.A.M. Versteijne
De toepassing en toekomst van public private partnership's bij de
grote en middelgrote Nederlandse gemeenten
- 320 Th. van de Klundert
Wage Rigidity, Capital Accumulation and Unemployment in a Small Open
Economy

Bibliotheek K. U. Brabant



17 000 01065929 1